P. A. Novikov and É K. Snezhko

UDC 536.422 .4

Results are shown of a study concerning the distribution of solid condensate on a cold cylinder surface under a vapor-gas pressure within the $10<p_{S}<10^{3} \mathrm{~N} / \mathrm{m}^{2}$ range.

Vacuum condensation of vapor on the outside cold surface of a cylinder is a technique often used in modern sublimation apparatus, in cryogenic and adsorption-condensation pumps for the purpose of scavenging vapor out of a vacuum system, also in distillation and in producing certain organic compounds which are unstable at high temperatures, etc.

The ever growing interest in these applications is not matched by the amount of data available and, therefore, requires further research.

It is well known that the buildup rate of solid condensate and its distribution over a cold surface are both determined by the thermodynamic properties of the medium ( $\mathrm{T}_{\mathrm{c}}, \mathrm{p}_{S}$ ) and on the hydrodynamics of its flow, largely dependent on the geometry of the condensation surface and its disposition relative to the divergent vapor stream [1-3].

The authors have succeeded in establishing [4] that the angle of divergence of a water vapor stream flowing from the sublimating body into the ambient medium, if the medium is a viscous or molecularviscous gas, and the profile of the solid condensate layer forming on a flat surface depend mainly on the ambient vapor-gas pressure. These facts may serve as the basis for a most appropriate design of flat sublimator and condenser surfaces inside a vacuum chamber.

The radial variation of local density from the center of mass to the periphery of a stream above a flat surface will serve as the point of departure in our analysis of the kinetics of vapor condensation on the outside cold surface of a circular cylinder. On the basis of evaluated test data for flat and cylindrical surfaces, we will then determine the optimum active surface of a vacuum condenser.

The condensation tests on a cylinder surface were performed under steady-state conditions in a thermostaticized vacuum chamber 1 (Fig. 1). On model VLTK-500 scales 12 was placed a condenser consisting of an aluminum tube $20.045 \pm 0.0025 \mathrm{~m}$ in diameter and 0.3 m long, whose wall was cooled across the gap from a coaxial inner cylinder $30.03 \pm 0.0025 \mathrm{~m}$ in diameter. Cylinder 3 was rigidly fastened to the wall of a pressure chamber and cooled by pumping through it liquid nitrogen. The length of the active condenser surface was by one order of magnitude larger than the diameter $D_{V}$ of the evaporation surface, while the distance between both surfaces did not exceed $2.5 \mathrm{D}_{\mathrm{V}}$, so that the stream of water vapor diverging from the sublimator covered the center portion of the tube. Edge effects could be disregarded here and the condensation process could be considered occurring at an infinitely long cylinder surface.

In order to increase the radiative component of the thermal flux through the annular channel between outer and inner cylinder, the opposite surfaces of both cylinders across the gap were covered with thin layers of lampblack, after thermocouples had been installed on them. The active surface of condenser 2 was graduated for measuring the area covered by condensate, while the condensate layer thickness along the cylinder generatrix was determined both by visual inspection (with a ruler) and by means of a recording instrument 5.

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Fig.1. Schematic diagram of the test apparatus.

The temperature field in the space between sublimator and condenser was measured with a thermocouple system consisting of ten shielded copper - constantan pairs held together with a thin polyamide thread 9, a multiposition stepper 8 with an electromagnetic switch, a low-resistance potentiometer, and a mirror galvanometer. Thermocouples frozen into the condensate at the condenser surface under the center of the vapor stream were spaced closest ( 2 mm apart), for a precise study of the temperature-time characteristics of the condensate layers buildup.

A continuous steady stream of water vapor striking the cold cylinder surface was produced by the sublimation of ice forming in the capillaries of a porous titanium plate 10 constantly supplied with distilled water. The water flow rate was measured by the volumetric method which had been described earlier in [4], while the sublimation rate was regulated with an electric heater 11 as well as by a flat protective heater, and a differential thermocouple.

The temperature of the condensation surface was maintained constant during the experiment by regulating the supply of coolant (liquid nitrogen) to the stationary cylindrical nitrogen shield. The heat transfer through the thin ( $\sim 0.005 \mathrm{~m}$ ) annular interlayer of rarefied gas was here purely conductive and radiative.

The dynamics of condensate buildup and distribution over the cylinder surface at the center of the vapor mainstream was studied with the aid of a special device consisting of a pantograph, a rolling wheel 4 , and a recording pen 5 which copied the condensate profile on millimeter paper. A model DSM-P-220 electric motor 7 with a speed reducer 6 was switched on during the test at equal time intervals and the profile of the building up desublimate layer was traced with the rolling wheel.

The entire assembly was installed on scales and, thus, the condensation rate was being measured continually throughout the testing time. It has been established, in this way, that for either a flat or a cylindrical condenser $J_{C}=f\left(p_{S}, T_{c}\right)$ and that the dimensionless parameter $\beta=\mathbf{f}\left(\mathrm{p}_{\mathrm{S}}, H\right)$ characterizing the profile of solid condensate is almost independent of the process time, with $p_{s}, T_{c}$, and $H$ constant, i.e., that the absolute quantity of condensate increases with time but the relative quantities on the condenser surface elements (the condensate profile) remain almost unchanged. All this holds true for a small layer where the distribution of condensate is not yet much affected by some temperature rise at the condensation surface. Our test data are in complete accord with the First Law of Mass Effect [5]. We were also able to connect the ball joints of the pantograph with the spring scale of a calibrated dynamometer and to evaluate, by the depth of the wheel track, the density of the superficial condensate layer forming on the cylinder surface during the experiment.


Fig. 2. Condensate distribution: on a cylinder (1 and 2 ), on a plate (3).

The test results have shown that a decrease of the ambient pressure causes the density of the building up condensate to increase appreciably. The superficial layer of condensate appears most brittle and vapor-permeable, especially under pressures near the "triple point."

The thermophysical and the mechanical properties of the condensate change appreciably in this layer, owing to the high rate of heat and mass transfer between desublimate crystals. As to the density, the specific heat, and the effective thermal conductivity of the forming solid condensate, we refer to the mean-over-the-thickness values in the building up layer, inasmuch as they are functions of the condensation time.

Since these values vary also over the entire phase-transformation surface, hence difficulties in an analytical solution of the problem will arise, of course, when vapor condensation under vacuum is considered and the experimental method of solution will, necessarily, become crucial.

In order to design the size and the shape of an active condenser surface and to determine the local condensation rates, it is particularly worthwhile to know how the distribution of vapor density within the volume of the vacuum chamber will vary with the pressure $p_{S}$.

The proposed method has made it possible to establish this relation for the case of a viscous or a molecular-viscous flow of water vapor, on the basis of the ice distribution on cold surfaces of various shapes. The effective solid angle $2 \varphi_{e}$ of the divergent vapor stream was determined during an analysis of condensation on an infinitely large flat plate, according to the procedure in [4]. It has been thus established that, under thermodynamic conditions characteristic of operating sublimators, the magnitude of this angle depends on the ambient vapor-gas pressure: $\varphi_{\mathrm{e}}=\cos ^{-1} \exp \left(-\mathrm{p}_{\mathrm{S}} / \mathrm{p}_{0}\right)$ when $0.01<\mathrm{p}_{\mathrm{S}} / \mathrm{p}_{0}<0.2$ and $\mathrm{T}_{\mathrm{c}}$ $=230-240^{\circ} \mathrm{K}$, but $\varphi_{\mathrm{e}}=\cos ^{-1} \exp \left(-0.42 \sqrt{\mathrm{p}_{\mathrm{S}} / \mathrm{p}_{0}}\right)$ when $0.2<\mathrm{p}_{\mathrm{S}} / \mathrm{p}_{0}<1$. Under these conditions the radial variation of layer thickness within the flat spot of condensate is best described by the equation of the Gauss curve [1]:

$$
\begin{equation*}
\frac{\Delta h_{R}}{\Delta h_{0}}=\exp \left[-\left(\frac{2 R}{D_{\mathrm{v}} \beta}\right)^{2}\right] \tag{1}
\end{equation*}
$$

with $\beta=0.55\left(1+2 H \tan \varphi_{\mathrm{e}} / \mathrm{D}_{\mathrm{V}}\right)$, i. . .,$~ \beta=\mathrm{f}\left(\mathrm{H}, \mathrm{p}_{\mathrm{S}}\right)$.
This empirical relation (1) is analogous to the approximate distribution function with the correlating statistical index of mass effect equal to $R$ in [5].

Disregarding the variation of the condensation factor and of the condensate density along radius $R$, but applying the law of cosines to molecules striking an inclined surface, we use the distribution equation (1) for determining how the effective density of the mass stream varies from the center to the periphery within the confines of the solid angle $2 \varphi e$ :

$$
\begin{equation*}
\frac{J_{0}}{J_{\varphi}}=\cos \varphi \exp \left[\left(\frac{2 R}{D_{\mathrm{v}} \beta}\right)^{2}\right] \tag{2}
\end{equation*}
$$



Fig. 3


Fig. 4

Fig. 3. Flow pattern of a vapor-gas mixture at a cylindrical surface.
Fig. 4. Variation in the optimum geometry of an active condenser surface, following an increase in the divergence angle of the vapor stream $\varphi=f\left(p_{S}\right)$ and an increase in the distance from the vapor source $H$.

For a cylinder with surface elements at an angle to the direction of the vapor-gas flow, the profile along the center cross section is

$$
\begin{equation*}
\frac{h_{0}}{h_{\gamma}}=\exp \left[\left(\frac{2 R_{\mathrm{cy} 1} \sin \gamma}{D_{\mathrm{v}} \beta}\right)^{2}\right] \frac{\cos \varphi}{\cos (\gamma+\varphi)} \tag{3}
\end{equation*}
$$

with angle $\varphi$ expressed in terms of angle $\gamma$ :

$$
\operatorname{tg} \varphi=\frac{R_{\mathrm{cyl}} \sin \gamma-D_{\mathrm{v}} / 2}{\dot{L}-R_{\mathrm{cyl}} \cos \gamma}
$$

and, after the appropriate transformation,

$$
\begin{equation*}
\frac{h_{0}}{h_{\gamma}}=\exp \left[\left(-\frac{2 R_{\mathrm{cyl}} \sin \gamma}{D_{\mathrm{v}} \beta}\right)^{2}\right] \frac{L-R_{\mathrm{cyl}} \cos \gamma}{L \cos \gamma+R_{\mathrm{u}} \sin \gamma-R_{\mathrm{cyl}}} \tag{4}
\end{equation*}
$$

A comparison in Fig. 2 between profile 1 based on Eq. (4) and profile 2 based on experiment with a cylindrical condenser surface under the same conditions indicates some decrease in the thickness of profile 2 near the condensate from under larger angles $\gamma$. An explanation for this ought to be sought, apparently, in the difference between the hydrodynamics of a stream at a flat and at cylindrical cold surface respectively.

It is necessary, therefore, to further study the hydrodynamic conditions prevailing at condensation surfaces of various shapes.

The pattern of a transverse vapor-gas stream flowing at a velocity of $0.1-0.2 \mathrm{~m} / \mathrm{sec}$ around an immersed circular cylinder has been visually observed and then photographed, as shown in Fig. 3. A typical feature of this flow pattern is the buildup of a vapor-gas cloud near the condensation surface from a minimum at $\gamma=0$ to a maximum at $\gamma=2 \pi$. This vapor-gas cloud represents an additional barrier to molecules and molecule complexes of vapor trying to penetrate to the cold cylinder surface - a resistance which increases directly with the angle $\gamma$. This also results in a less uniform distribution of solid condensate (profile 2 in Fig. 2) around the cylindrical condenser ( $\mathrm{R}_{\mathrm{cyl}} / \mathrm{D}_{\mathrm{V}}=0.75, \mathrm{~L} / \mathrm{D}_{\mathrm{V}}=3.25$ ). For comparison, we show in Fig. 2 the distribution of condensate forming under the same thermodynamic conditions on a flat cold surface (profile 3). Thus, from the laws of condensate distribution on condenser surface elements we have established the distribution of vapor density. Solving now the reverse problem, from an already empirically known radial distribution of vapor density from the mainstream center to the periphery we can design an active condenser surface whose every element will form a respective angle $\alpha$ with the sublimation surface and which will be located at a given distance from it. These angles and distances must ensure a uniform buildup of the condensate layer on the designed surface.

Let the mean layer thickness on a flat surface $\pi D_{c}^{2} / 4$ be the optimum. Then for (1) we have

$$
\begin{equation*}
\frac{h_{\mathrm{m}}}{h_{0}}=\left(\beta D_{\mathrm{v}} / D_{\mathrm{c}}\right)^{2} . \tag{5}
\end{equation*}
$$

This relation remains invariant with constant $\mathrm{p}_{\mathrm{S}}$ and H .
It is easy to see that

$$
\begin{equation*}
\operatorname{tg} \alpha=\sqrt{h_{R}^{2}-h_{\mathrm{m}}^{2}} / h_{\mathrm{m}} \tag{6}
\end{equation*}
$$

Inserting here (1) and (5), we obtain, after integration,

$$
\begin{equation*}
\Phi(R)=\int\left\{\left(\frac{D_{c}}{\beta D_{\mathrm{v}}}\right)^{4} \exp \left[-\left(\frac{2 \sqrt{2} R}{\beta D_{\mathrm{v}}}\right)^{2}\right]-1\right\}^{1 / 2} d R \tag{7}
\end{equation*}
$$

The resulting function (related to the error probability) within the interval $0<R<R_{m}$, where $R_{m}=\left(\beta D_{v}\right.$ /2) $\left[\ln \left(\mathrm{h} / \mathrm{h}_{\mathrm{m}}\right)\right]^{1 / 2}$, defines exactly the curve which, by rotation about the symmetry axis of the stream, will yield the desired condensation surface based on the criterion of uniform condensate thickness $\mathrm{h}_{\mathrm{m}}$ buildup. The solid lines in Fig. 4 indicate segments of such axially symmetric surfaces and the trend in their variation with an increasing angle of vapor stream divergence from $\varphi_{1}=19^{\circ}$ to $\varphi_{2}=46^{\circ}$ (such an increase follows a rise in pressure $p_{S}$ from 33 to $465 \mathrm{~N} / \mathrm{m}^{2}$ ) as well with an increasing distance between these surfaces and the evaporation surface $H / D_{V}$ from 1 to 3 . The upper parts of these segments may be disregarded in our case, because uncondensed gas cannot be easily removed from such surfaces after extended process periods and the condensation surfaces become very distorted then.

The lower parts of these segments have the drawback of having been plotted without consideration of the hydrodynamics of vapor flow near a body of such a complex shape. Correcting the orientation of such a surface to account for the flow hydrodynamics (within the same interval $0<R<R_{m}$ ), therefore, will yield the optimum condensation surface indicated in Fig. 4 by symmetric segments (dashed-dotted lines). These curves almost coincide with the streamlines plotted according to the equation in [1]:

$$
\begin{equation*}
\psi=-\left(\frac{z+a}{\sqrt{r^{2}+(z+a)^{2}}}+\frac{z-a}{\sqrt{r^{2}+(z-a)^{2}}}\right), \tag{8}
\end{equation*}
$$

and derived for a theoretical design of the conical guide vane in a vacuum pump by the Chaplyagin method (the hydrocone problem). In Eq. (8) the streamline $\psi=$ const. It follows from the experiment in [4] that the magnitude of $\psi$ depends essentially on the density of the ambient medium and that the coefficient $a$ is proportional to the distance $H$ from the sublimation surface; $z$ and $r$ are space coordinates.

In the design of active surfaces for vacuum condensers operating under typical industrial sublimator pressures, however, one must not only consider a uniform distribution of condensate on the cold element and ensure a continuous removal of uncondensed gas from the condenser surface, but one must also consider as crucial the technical feasibility of producing such a surface. On this basis, and allowing for changes in the geometry of the entire condensation surface during the buildup of new condensate layers, we have arrived at the concept of a toroidal condenser surface with a flat vapor source. Such a surface is generated by rotating a circular arc with radius $\mathrm{R}_{\mathrm{m}}$ (dashed lines in Fig. 4) about the tangent to it which coincides with the symmetry axis of the vapor stream, with $R_{m}$ and $H$ determined by the thermodynamics of the condensation proces. More precisely, only part of this surface is active: the part subtended by the effective solid angle $2 \varphi_{\mathrm{e}}$ of the stream cone.

For orifice-type sublimation sources used in practice, one follows an analogous procedure in determining the geometry of the condenser surface close to the sublimator orifice in the vapor stream. Such a surface can be produced by piecing together concave surface elements of a circular cylinder whose common generatrix lies in the symmetry plane of the vapor stream from the orifice source and becomes the front edge protruding into the oncoming vapor stream.

In this case, for maintaining a constant and uniform (over the condensation surface) temperature, it is recommended that such a cold element be produced like a cryogenic heat exchanger pipe with external fins made up of these concave cylindrical surfaces. If these tubular elements are now spaced in a row above the vapor source, and if another row follows behind in a staggered position, then the number of necessary such stages will be governed by the condensation factor characterizing the given thermodynamic
conditions under which the sublimator operates. Thus, a uniform freezing of solid condensate on cold surfaces can be ensured together with an efficient utilization of the active volume in the vacuum chamber.

## NOTATION

$\mathrm{p}_{0}, \mathrm{p}_{\mathrm{s}}$
$\mathrm{T}_{\mathrm{c}}$
$\mathrm{J}_{0} / \mathrm{J}_{\varphi}$
$\mathrm{h}_{0}, \mathrm{~h}_{\mathrm{R}}, \mathrm{h}_{\mathrm{m}}$
H
$\mathrm{R}_{\mathrm{cyl}}$
$\mathrm{L}=\mathrm{H}+\mathrm{R}_{\mathrm{cyl}} ;$
$\mathrm{D}_{\mathrm{V}}, \mathrm{D}_{\mathrm{c}}$
$\varphi_{\mathrm{i}}, \gamma_{\mathrm{i}}$
$\alpha$
$\beta$
are the "triple point" pressure and pressure inside the chamber, respectively; is the temperature of the condensation temperature; is the relative density of diverging vapor stream; are the thickness of condensate layer at the center of the mainstream, at distance R from the mainstream, and the mean thickness (at $R_{m}$ ), respectively;
$\mathrm{R}_{\mathrm{cyl}}$
$\mathrm{L}=\mathrm{H}+\mathrm{R}_{\text {cyl }}$;
$D_{V}, D_{c}$
$\varphi_{\mathbf{i}}, \gamma_{\mathbf{i}}$
$\alpha$
$\beta$ is the distance from sublimator to condenser; is the radius of cylinder;
are the diameter of flat evaporation surface and of flat condensation surfaces, respectively;
are the angular space coordinates; is the angle between condensation surface and sublimation surface ensuring a uniform buildup of mean layer thickness, $\mathrm{h}_{\mathrm{m}}$;
is the parameter characterizing the condensate profile, on a flat surface.

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